FREE CONVECTION OF NONLINEARLY VISCOUS FLUIDS AROUND THIN BODIES OF REVOLUTION UNDER BOUNDARY CONDITIONS OF THE SECOND KIND

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The problem of the free convection of a "power-law" fluid around thin bodies of revolution under boundary conditions of the second kind (q = const) is studied. The influence of the transverse body curvature and the non-Newtonian behavior parameter on the heat exchange and friction is determined.

Problems of the free convection of nonlinearly viscous fluids under boundary conditions of the first kind (principally for a nonisothermal wall) have been examined in a whole series of papers, of which [1, 2] can be noted. However, physical situations when the heat flux on the wall rather than the temperature is known or given, i.e., boundary conditions of the second kind are realized (for example, in the electrical heating of surfaces), are encountered most often in applications. Three-dimensional problems of the free convection of "power-law" fluids around thin bodies of revolution (the boundary layer thickness is commensurate with the radius of the body of revolution) under boundary conditions of the second kind have not yet been analyzed. Such problems are often encountered in applications, especially in the technique of probe measurements, in needle and cylindrical automatic regulation transducers, and remote control of technological processes.

As a rule the generalized Pr number is quite large [3] for non-Newtonian fluids. Consequently, the thickness of the thermal boundary layer turns out to be much less than the dynamic boundary layer thickness. Hence, the contribution of inertial terms within the thermal boundary layer can be neglected. Then the dimensionless boundary layer equations for the problem of free convection around a thin vertical body of revolution are

$$\frac{\partial}{\partial y}\left[r\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right]+r\Theta\cos\alpha_1=0, \tag{1}$$

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0, \tag{2}$$

$$ru\frac{\partial\Theta}{\partial x}+rv\frac{\partial\Theta}{\partial y}=\frac{\partial}{\partial y}\left[r\frac{\partial\Theta}{\partial y}\right]. \tag{3}$$

The boundary conditions are

$$u=v=0,\quad \frac{\partial\Theta}{\partial y}=-1\quad \text{for}\quad y=0,$$

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$$\frac{\partial u}{\partial y}\to 0,\quad \Theta\to 0\quad \text{for}\quad y\to\infty \tag{4}$$

(the temperature gradient on the wall is chosen negative; however, the results of the research are easily transferred to the case of positive gradients). For thin bodies of revolution $\alpha_1 \approx 0$ ($\cos\alpha_1 \approx 1$). The introduction of new dependent and independent variables and parameters

$$\xi=\int_0^x U^{\alpha}(x)\, r_0^{\beta}(x)\, dx,\quad \eta = a\xi^{-\gamma}U(x)\int_0^y r\,dy,$$
$$\Psi = b\xi^{\gamma} f(\eta),\quad \Theta = c\xi^{\nu} g(\eta),\quad A=\frac{2\xi^{\gamma}}{aU(x)\, r_0^2(x)}, \tag{5}$$

where Ψ is determined from the continuity equations

$$ru=\frac{\partial\Psi}{\partial y},\quad rv=-\frac{\partial\Psi}{\partial x},$$

and the requirement of self-similarity of the system of equations obtained from (1)-(4) with (5) taken into account, results in the following equations:

$$\frac{\nu}{\gamma}=\frac{n}{3n+2},\quad \beta=\frac{1-\gamma+(2\nu-\gamma)\alpha}{\nu}, \tag{6}$$

$$U=x^{\frac{n+2}{3n+2}},\quad r_0 = mx^{\frac{n}{3n+2}},\quad \Theta = c\gamma^{\nu} m^{\nu\beta} x^{\frac{n}{3n+2}} g(\eta). \tag{7}$$

In order to eliminate the constants from the final system of differential equations, it is necessary to require

$$a=m^{\beta\gamma-1}\gamma^{\nu},\quad b=m^{1-\beta\nu}\gamma^{-\nu},\quad c=m^{-\beta\nu}\gamma^{-\nu}. \tag{8}$$

The system of relationships (6) admits of two degrees of freedom for the selection of α, β, γ, and ν. This affords the possibility of giving one quantity from the pairs of parameters γ and ν, α and β, arbitrarily. The quantities a, b, and c are determined from (8) depending on this selection. However, the final system of differential equations has a universal form relative to these constants

$$n(1+A\eta)^{\frac{n+1}{2}} f'''|f''|^{n-1}+\frac{n+1}{2}(1+A\eta)^{\frac{n-1}{2}} A|f''|^{n-1}f''+g=0,$$
$$(1+A\eta)g''+Ag'-\frac{n}{3n+2}gf'+g'f=0 \tag{9}$$

with the boundary conditions

$$f=0,\ f'=0,\ g'=-1\quad \text{for}\quad \eta=0,$$
$$f''\to 0,\quad g\to 0\quad \text{for}\quad \eta\to\infty. \tag{10}$$

There results from (7) and (8) that the wall temperature should vary according to the power law

$$\Theta|_{y=0}=g(0)\, x^{\frac{n}{3n+2}}. \tag{11}$$

The local heat exchange and friction coefficients are determined from the formulas

$$\mathrm{Nu}=g^{-1}(0)\mathrm{Gr}^{\frac{1}{n+4}}\mathrm{Pr}^{\frac{n}{3n+2}}x^{\frac{2(n+1)}{3n+2}},$$
$$c_f = 2[f''(0)]^n\,\mathrm{Gr}^{-\frac{1}{n+4}}\mathrm{Pr}^{\frac{4}{3n+2}}x^{\frac{2n}{3n+2}} \tag{12}$$

TABLE 1. The Values of $f''(0)$ and $g(0)$

A	0,05	0,5	1,5	3	5
			$n=1$		
$f''(0)$	1,437	1,422	1,422	1,456	1,517
$g(0)$	1,510	1,328	1,068	0,8477	0,6792
			$n=0,75$		
$f''(0)$	1,647	1,636	1,661	1,742	1,868
$g(0)$	1,514	1,332	1,072	0,8512	0,6822
			$n=0,50$		
$f''(0)$	2,121	2,131	2,212	2,438	2,742
$g(0)$	1,505	1,325	1,072	0,8492	0,6814
			$n=0,25$		
$f''(0)$	3,888	4,228	5,111	7,468	13,40
$g(0)$	1,428	1,225	0,9700	0,7346	0,5439

The average coefficients respectively equal

$$\overline{\mathrm{Nu}} = \frac{3n+2}{5n+4}\, g^{-1}(0)\,\mathrm{Gr}^{\frac{1}{n+4}}\,\mathrm{Pr}^{\frac{n}{3n+2}},$$
$$\overline{c_f} = \frac{2(3n+2)}{5n+2}\,[f''(0)]^n\,\mathrm{Gr}^{-\frac{1}{n+4}}\mathrm{Pr}^{\frac{4}{3n+2}}. \qquad (13)$$

The system of equations (9) with the boundary conditions (10) was solved numerically on a "Minsk-22" electronic computer by a modified Newton method. Some of the computation results are presented in the table. An increase in the values of the transverse curvature parameter A results in a significant reduction in the wall temperature and, therefore, to a rise in both the local and average heat-exchange coefficients according to (12) and (13). Let us note the slight influence of the transverse curvature parameter on the friction characteristic $f''(0)$ for Newtonian and slightly pseudoplastic media and the stronger influence for very pseudoplastic media. The quantity $f''(0)$ increases sharply for fixed values of the parameter A as the pseudoplasticity increases. Attention is turned to the interesting fact of the quite slight influence of the exponent of non-Newtonian behavior n on the quantity g(0) for fixed values of the transverse curvature parameter. However, although the quantity g(0) depends slightly on n, the wall temperature and the heat-exchange coefficient depend strongly on n as (11)-(13) show. The results of the research are extended easily to the case of concentration free convection (in electrolyte solutions, for instance).

NOTATION

$x = \frac{x'}{L}$, $y = \frac{y'}{L}\,\mathrm{Gr}^{\frac{1}{n+4}}\mathrm{Pr}^{\frac{n}{3n+2}}$, $r = \frac{r_0'+y'}{L}\,\mathrm{Gr}^{\frac{1}{n+4}}\mathrm{Pr}^{\frac{n}{3n-2}}$ are dimensionless coordinates;

$u = u'\mathrm{Pr}^{\frac{n+2}{3n+2}}\mathrm{Gr}^{\frac{1}{2(n+4)}}\left[\frac{L^2 g\beta q_0}{\lambda}\right]^{-\frac{1}{2}}$, $v = v'\mathrm{Pr}^{\frac{2(n+1)}{n+2}}\mathrm{Gr}^{\frac{3}{2(n+4)}}\left[\frac{L^2 g\beta q_0}{\lambda}\right]^{-\frac{1}{2}}$ are dimensionless velocities;

$\Theta = \frac{(T-T_\infty)\lambda}{q_0 L}\,\mathrm{Gr}^{\frac{1}{n+4}}\mathrm{Pr}^{\frac{n}{3n+2}}$ is the dimensionless temperature; x' and y' are dimensional coordinates; r_0' is the dimensional body radius; u' and v' are dimensional velocities; L is the characteristic dimension; α_1 is the angle between the tangent to the body outline and the direction of gravity; T_∞ is the absolute temperature as $y \to \infty$; q_0 is the constant heat flux to the wall; λ is the heat conductivity, $\mathrm{Pr} = \left(\frac{\rho}{k}\right)^{-\frac{5}{n+4}} \frac{\rho c_p}{\lambda} L^{-\frac{2(n-1)}{n+4}} \times \left(\frac{\beta g q_0}{\lambda}\right)^{\frac{3(n-1)}{n+4}}$ is the modified Prandtl number; $\mathrm{Gr} = \left(\frac{\rho}{k}\right)^2 L^4 \left(\frac{\beta g q_0}{\lambda}\right)^{2-n}$ is the modified Grashof number; β is the coefficient of volume expansion; g is the free-fall acceleration; k is the coefficient of consistency; n is the non-Newtonian behavior parameter; Ψ is the modified stream function; r_0 is the radius of the body of revolution; η, $f(\eta)$, $g(\eta)$ are self-similar variables; A is the curvature parameter; Nu and c_f are the local Nusselt number and friction coefficient; $\overline{\mathrm{Nu}}$ and $\overline{c}_f$ are the average Nusselt number and friction coefficient.

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